REVISITING TRANSFER OF LEARNING IN MATHEMATICS: INSIGHTS FROM AN URBAN LOW-INCOME SETTLEMENT

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Debates on transfer of learning in mathematics are not new. Claims of situatedness of learning within tasks and non-transferability of knowledge between tasks are widely contested. Direct Application (DA) of learning is a common paradigm for characterising transfer which led to many instances of transfer failure (Bransford & Schwartz, 2001). The alternative framework presented here shows that some of the transfer failures in mathematics can be considered as partial transfer by broadening the DA paradigm which can scaffold classroom pedagogy by drawing upon everyday mathematics. Claims are supported by data drawn from an economically active low income settlement where sample middle graders are engaged in house-hold based micro-enterprise, possess diverse opportunities for gaining mathematical knowledge.

THE CONTEXT

Debates on situated perspective and transfer of learning in mathematics are not new. There have been several arguments in favour of and against learning transfer. Beginning with the pioneering work of Lave (1991), Lave and Wenger (1991) and Greeno, Smith and Moore (1992) situated learning in the domain of mathematics education broadly claimed that learning is situated within tasks at hand and that knowledge is non-transferable between different tasks. But, Anderson, Reder and Simon (1996) contested by arguing that such claims are "sometimes inaccurate and exaggerated", and the "implications drawn are mistaken" (p. 5, 6). Anderson *et al*.'s contention (*ibid*) was further challenged by Greeno's (1997) objections of the generality and presuppositions about the levels of analysis that Anderson *et al*. had adopted. Greeno argued that the counterclaims of Anderson *et al*. addressed different questions by focusing on "knowledge and contexts of performance" and not the "activities and situations in which activities occurred and learned" (p. 6) and therefore they answered wrong questions. Earlier debates on transfer of learning indicate a growing feeling within cognition researchers about too many instances of transfer failure and lack of evidence that can challenge Thorndike's assertion that transfer is rare and occurs only between two similar situations (Bransford & Schwartz, 2001). Transfer literature is thus based on claims and counterclaims that looked at different notions of transfer devoid of unanimity over any concrete outcome. Most debates came within the paradigm of "direct application" (DA) of learning to new problem situations often following "sequestered problem solving" (SPS) method (Bransford & Schwartz, 2001). In this paper, it is argued that sticking to the rigid boundaries of DA paradigm could be one reason for many instances of transfer failure and that we need a broader perspective to incorporate the doer's goal structure while using algorithms and learning

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the underlying principles in order to look at the transfer phenomenon holistically. The broader perspective on transfer can then help us address educators' concern for ensuring effective learning among students and increase their ability to carry forward such learning. DA and SPS characterisations detect transfers with a “yes-no”, or “either-or” result and fail to indicate occurrences of partial transfer which can actually prepare ground for future learning. Transfer in everyday settings seldom leads to black and white conclusions. In this paper, an alternative framework is employed to revisit the transfer problem and reject the previously held characterisation of transfer as only direct application (DA) and also Lave's claim of non-transferability of knowledge. Extending and partially revisiting Bransford and Schwartz' notions of transfer, the alternative framework (*Algorithm Goal Structure*) addresses the transfer problem by considering comprehension and use of arithmetical algorithms as the central goal followed by learning to apply or relate to the underlying principles implicitly or explicitly.

Drawing from data collected through interviews and discussions with working middle graders from an economically active urban low income settlement, it is observed that children's work-contexts are diverse and consequently, the extent and type of mathematical knowledge that students acquire outside school can be expected to show diversity. Such diverse engagement with contexts help children develop effective context specific problem solving ability that could be used for effective mathematics learning in the classrooms (Bose & Subramaniam, 2013). In this paper, transfer of learning is explored among the sample middle graders while they solve mathematical problems reflecting everyday contextual situations in the school set up as well as in the situations that emerge in the work contexts. It is claimed that the occurrence of partial transfer works as scaffolds for better learning of different components of the algorithms and principles, unlike Bransford and Schwartz's relatively vague and indefinite notion of “preparation for future learning” (p. 69).

TRANSFER OF LEARNING (IN MATHEMATICS)

Different paradigms looked at transfer of learning and common among them was the direct application (DA) paradigm based on the notion of “initial learning followed by problem solving”. Bransford and Schwartz (2001) however moved from direct application of knowledge to the “perspective of preparation for future learning”. This notion is in opposition to those that Lave, Anderson *et al*. or Greeno had adopted. Table 1 below highlights the transfer notions that prominent researchers adopted.

Claims for both successes and failures in achieving learning transfer came up due to inconsistencies prevalent in the way transfer was defined. It is pertinent therefore for the educational researchers to revisit the definition and make generalisable claims that can be used for drawing larger pedagogic pointers for effective mathematics learning. Lave and Wenger (1991) looked at learning in the processes of co-participation as a situated activity, focusing on skill acquisition through engagement in tasks and claimed that situated perspective demonstrated that skills (or action) grounded in tasks

often did not "generalise to school situations". In contrast, Anderson *et al*. argued that closer analyses of the tasks were required to make tenable claims of non-transferability of learning (1996, p. 6) and demonstrated situations where learning transfer occurred across contexts by showing transfer of mathematical competence from classroom situations to laboratory situations.

Thorndike's definition	Whether people can apply their knowledge to new a problem or situation (1901, as cited in Bransford & Schwartz, 2001)
Lave's definition	Transferring one's knowledge and skills from one problem-solving situation to another (1988)
Anderson *et al.*'s & Greeno's definition	Not explicitly defined, used prevalent notion of direct application
Bransford & Schwartz's definition	Moved from Direct application of knowledge (DA) to Preparation for future learning (PFL) (2001)

Table 1: Definition of transfer used by prominent researchers

Everyday mathematics vs School mathematics

"Everyday mathematics" is considered here as a form of mathematics used in out-of-school settings while engaging in contextually embedded practices. Everyday and school mathematics describe two forms of activities based on the same mathematical principles but on different cultural practices (Nunes *et al*., 1993). While the former offers freedom of using alternate techniques than those learnt at school, the latter is more of symbol based often detached from meaningful contexts. Distinction between everyday and school math can be summed up by contrasting their *basic features* (group-work, division of labour vs individual, independent work; tool manipulation vs pure mentation), *goals* (situation specific competencies vs generalised learning), *difference in procedures* (orality vs written, use of multiple units, contextualised reasoning vs use of symbols and formal reasoning), *difference in mechanisms of knowledge acquisition* (sharing, communication vs textbook based knowledge), and *metacognitive awareness* (meaningfulness, continuous monitoring vs algorithm-use, lack of meaningfulness, relevance) (Nunes *et al*., 1993; Lave and Wenger, 1991; Resnick, 1987; Saxe, 1988). We make use of the everyday mathematics framework to locate the transfers of learning as they emerge in the classroom practices or in the everyday settings, in routine work-contexts, or during economic transactions.

THEORETICAL FRAMEWORK: ALTERNATIVE VIEW OF TRANSFER

The whole debate depends upon what counts as transfer. In the alternative framework of *Algorithm Goal Structure* (AGS), "direct application" is used as the first filter and Bransford and Schwartz' definition of "*preparation for future learning*" as the second filter. The first filter considers definition of transfer on the following criterion:

- Have students learned the algorithms?
- If they have learned the algorithms, whether they can apply the underlying principles implicitly or explicitly?

In the second filter, it is checked whether students are able to transfer elements from their everyday or school knowledge in terms of some components of the underlying principles or the algorithms. In the present analysis, Bransford and Schwartz' notion of "*preparation for future learning*" is restricted to the possibility of learning the algorithms and elements of everyday knowledge that contribute to the components.

SAMPLE & METHODS

The sample for the larger ethnographic study done over two years was drawn from grade 6 of the municipality-run English and Urdu medium schools located in a low income settlement in Mumbai. This area has a vibrant economy in the form of house-hold based micro enterprise, which provide employment to the dense population living there. A total of 31 students were chosen randomly (every third student from the attendance register) to form the sample. Data was collected in three separate parts: the first part was semi-structured interviews of all 31 students to understand their family-background, socio-economic conditions, parental occupation, productive work done at home/elsewhere and student's involvement in them. The second part was interviews based on a structured questionnaire to understand students' basic arithmetical knowledge while the third part focused on students' knowledge about their work-contexts. All the interviews were audio recorded with prior permission from the respondents, the school authorities and also from the parents. The present data source is the second part of the interviews (arithmetical knowledge).

Location of the study

The large low-income settlement is located in central Mumbai where practically every house-hold is involved in income-generating work in which children take part from a young age. Being an old and established settlement, this low-income area attracts skilled and unskilled workers from all parts of India who come to the financial hub Mumbai in search of livelihood. The settlement is multi religious (Muslim majority) and multilingual (different language groups: Hindi/Urdu, Gujarati, Marathi, Tamil, Telugu). Common house-hold occupations include embroidery, *zari* (needle work with sequins), garment stitching, making plastic bags, leather goods (bags, wallets, purses, shoes), recycling work, etc. The goods produced here are sold not only in Mumbai but even exported, mainly to the Middle East countries.

INSIGHTS FROM THE FIELD

Transfer from everyday setting to school setting: Contextual problem-solving

Students' strategies while solving contextual problems presented in the school setting involved use of halving methods and convenient groupings that are commonly encountered in everyday setting. For example, while solving a school-type proportion

problem of finding the price of 25 *burfi* when 20 *burfi* cost 42 rupees, some students found the prices of 10 and 5 *burfi* by halving 42 and 21, *"bees ka bayalees, dus ka ikkis aur paanch ka gyarah*" [*forty two for twenty, twenty one for ten and eleven for five*]. However, not many students could do this way and opted for "unitary" method of finding the price of one *burfi* first and then that of 25, but eventually got stuck in the middle. Some students arrived at 53 as the answer and justified that sellers often do not return small change of Re 0.50. Most students (barring a few who used unitary method) could not figure out how to proceed and struggled to choose an arithmetical operation for solving the problem. Under the first filter of DA, transfer does not occur in the sense of using formal algorithm (unitary method) as a generalised technique. But, using the second filter, we notice that some students were able to use their everyday contexts and reality perspective in using halving technique that allowed them to find the price of the "difference" in the number of *burfis*. From pedagogic viewpoint, this is a scaffold for teaching the generalised requirement of finding the price of one *burfi*. Under AGS, it fits as a case of transfer through second filter.

In another problem (*finding the number of days 16 kerosene oil cans can last if one can lasts for 7 days*), most students used their everyday mathematical knowledge. For example, one student while computing orally, grouped 15 days for 2 cans and arrived at 4 months for 16 cans (considering 30 days per month) and then compensated the extra counts of 1 day per 2 cans, by subtracting 8 days and arrived at 112 days. Under the DA filter, one can claim that transfer is not happening if the algorithmic goal was to use formal multiplication. But, upon relaxing this goal, it fits as a case of transfer since the student could draw upon her everyday mathematical knowledge to solve the problem. Interestingly, only one student used the multiplication table of 16 while some students used the tables of 10 and 6 and added the partial products.

Many students found the formal algorithm for division difficult to use and they often use convenient strategies. For example, one student actually divided 315 by 5 presented symbolically on the paper and obtained 13 as the answer. He soon realised the error that the actual result cannot be that less. Subsequently, he aborted the formal division and did mental computation. Under DA filter this example does not show transfer but under the second filter, student's use of everyday experience emerges in realising the error and learning of repeated distribution as the underlying principle.

Use of fractions

Binary fractions like *aadha* (half), *paav* (quarter), *aadha-paav* (half-quarter, i.e., one-eighth) etc. are part of the everyday discourse that most students were exposed to and comfortable in using. The common contexts where binary fractions are used and which students regularly come across, are while buying provisions, vegetables, milk, etc. Non-routine fractions remain difficult for most students to comprehend and remains poorly developed, whereas binary fractions are easy for them to visualise and concrete visuals of whole numbers are easy to come by. Beyond these imagining and correlating divisions with numbers become difficult. The transfer of learning in case of binary fractions are only partial which fails at the DA filter since transfer does not

reach formalism but students are able to transfer some components of their everyday knowledge (fraction knowledge) which can scaffold learning of non-routine fractions.

Classroom activities[1]

During a classroom activity of shirt measurement which aimed at drawing upon students' everyday mathematical knowledge for informing classroom teaching, almost all the students preferred using *inch* tape (a popular measurement mode) for taking measurements although textbooks deal only with the international standard units. Most students however, did not know the relations between old British units (inch, foot) and standard international units (centimetre, metre). Students commonly used other indigenous measuring units like *bitta* (arm length) and *futta* (template used in tailoring work) during classroom discussions. AGS framework does not consider this example as transfer under DA since students could not convert two systems of units, but this example qualifies for partial transfer under the second filter on account of bringing in elements of everyday knowledge which can scaffold further learning.

Transfer from school setting to everyday setting: Problem-solving

In the work-contexts

Work-contexts of some students require doing quick calculations and use of approximation and estimation skills. Garment recycling work, for example, involves many children and requires weight measurement of the collected cloth pieces of varying size, colour and texture. The collected pieces are then sold off and the price is negotiated which requires children to make quick decisions and calculations. Children use convenient strategies and develop situation specific competencies, some of them reported use of multiplication tables that they learned at school. One student said that he does the multiplication "up in the air" by visualising the whole operation. He claimed that he does multiplication to cross-check the money he received.

Everyday shopping

School taught formal algorithms are often part of the daily routine, for example, at general stationary stores, sellers use paper and pen to arrive at the total cost. Oral computations are preferred while dealing with small amount of goods. Some students claimed that they cross-check the calculations on a paper using the school learned algorithms. Such examples indicate direct application of school taught methods and show transfer of learning to a different context. Number approximations during computations however qualify the second filter and shows partial transfer of learning.

Transfer failure

There are occasions where learning transfer did not seem to occur. It could be due to poor mathematical learning and lack of preparedness to handle complex calculations.

[1] The classroom activities are drawn from the vacation camp classes for the grades 6 and 7 students of the Urdu school that the researcher and his colleagues conducted. The activities do not reflect actual classroom teaching.

For example, when the researcher discussed with a student whether she was satisfied with the *Rakhi* making (decorative wrist-bands) wage, she answered in affirmation. Upon asking she could only tell the market price of one dozen *Rakhi* – at least Rs 60 (one *rakhi* is sold for Rs 5). She was unable to find the price at which one gross (twelve dozen) of *rakhi* is sold – Rs 720 for making of which she only earns Rs 15 or less. She could not calculate the amount she earns for making a single *rakhi* – which is about 10 paise (one-tenth of a rupee). She could neither use the school taught multiplication algorithm nor any other computation strategy. There was no reflection of the use of any form of mathematical knowledge – school or everyday. Transfer of learning from either context was not visible. One can also argue here that poor learning of school mathematics impedes workers like her from checking the fairness of a deal or the wage and entitlements that are distributed among the workers.

DISCUSSION & IMPLICATIONS

Interactions with students showed instances where transfer of learning occurred and where it did not occur. Transfer or non-transfer both emerged as instances of computation strategies and such instances are often connected with knowledge and skill acquisition that are valorised in the community. Economic, social and cultural practices of the households often influence children's learning of strategies to meet different needs, like optimal use of limited resources, management of house-hold chores, routine purchase of provisions, and so on. Gaining such traits are seen as essential in the community. Arguably therefore, low socio-economic conditions affect diverse skill acquisition and induce transfer achievement or partial transfer as seen among the sample students. Diverse work-contexts and everyday settings create affordances that support mathematics learning and they are important sites of learning transfer between school and everyday mathematics. From pedagogic viewpoint such potentially rich contexts offer strong foundation for effective mathematical learning.

The examples discussed above showed how rigid boundaries of DA would term many of them as transfer failures whereas many of these examples carried elements of everyday mathematical knowledge and some components of the underlying principles. Transfer failure often occurs not just due to the lack of exposure to everyday contexts but also on account of less conceptual mathematical reasoning and cognitive preparedness. Encumbrance of using formal algorithms is another possible reason. However, transfer failure occurring due to partly applying algorithms can scaffold learning of the underlying principles and achieve the algorithm goal, i.e., complete understanding of the procedure and the rationale. Thus, partial transfer emerging from the use of small components of the underlying principles has strong pedagogic relevance for better learning. From a pedagogic viewpoint these are pointers that can help the educators connect everyday mathematics with school mathematics. Hence, it was rather essential to look for possible instances of partial transfer and that required broadening of the DA filter. The proposed "Algorithm Goal Structure" (AGS) model looks at the instances that are potentially strong to work as scaffolds for effective learning. AGS matches with the growing consensus and with several educational

policy documents on bringing children's experience and prior knowledge to the classrooms and treating them as good starting points for building new knowledge. The arguments and claims made here are however evolving and calls for deeper exploration of their systemic and pedagogic underpinnings.

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